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**DATA.STAT.770 Dimensionality Reduction and Visualization, Spring 2025**

**Exercise set 2**

**Problem A1:**

a)

import numpy as np

import matplotlib.pyplot as plt

data = np.loadtxt('/Users/vudinhthi2304/Desktop/DATASTAT770/DATA.STAT.770-Dimensionality-Reduction-and-Visualization/Exercise2/week02/noisy\_sculpt\_faces.txt')

# Extract features and angles

features = data[:, :-3]

angles = data[:, -3:]

# Part a)

def nearest\_neighbor\_predictor(features, angles):

n = features.shape[0]

predictions = np.zeros\_like(angles)

for i in range(n):

# Compute the sum of squared differences

distances = np.sum((features - features[i]) \*\* 2, axis=1)

distances[i] = np.inf # Exclude the current face

# Find the nearest neighbor

nearest\_neighbor = np.argmin(distances)

predictions[i] = angles[nearest\_neighbor]

return predictions

Baseline leave-one-out error: 507129.623964493

b)

# Part b)

def leave\_one\_out\_error(features, angles):

predictions = nearest\_neighbor\_predictor(features, angles)

errors = np.sum((predictions - angles) \*\* 2, axis=1)

total\_error = np.sum(errors)

return total\_error

def forward\_selection(features, angles):

n\_features = features.shape[1]

selected\_features = []

min\_error = float('inf')

best\_features = None

errors = []

while True:

best\_feature = None

for feature in range(n\_features):

if feature in selected\_features:

continue

current\_features = selected\_features + [feature]

current\_error = leave\_one\_out\_error(features[:, current\_features], angles)

if current\_error < min\_error:

min\_error = current\_error

best\_feature = feature

best\_features = current\_features

if best\_feature is None:

break

selected\_features.append(best\_feature)

errors.append(min\_error)

print(f'Selected feature: {best\_feature}, Current error: {min\_error}')

return best\_features, min\_error, errors

best\_features, best\_error, errors\_list = forward\_selection(features, angles)

print(f'Best leave-one-out error: {best\_error}')

print(f'Number of features selected: {len(best\_features)}')

print(f'Selected features: {best\_features}')

Selected feature: 189, Current error: 310117.257272852

Selected feature: 53, Current error: 186791.79094106288

Selected feature: 137, Current error: 125693.66783699277

Selected feature: 224, Current error: 118213.72795439238

Selected feature: 32, Current error: 108774.43934657286

Best leave-one-out error: 108774.43934657286

Number of features selected: 5

Selected features: [189, 53, 137, 224, 32]

c)

# Part c)

def forward\_selection\_variant(features, angles):

n\_features = features.shape[1]

selected\_features = []

errors = []

for \_ in range(n\_features):

best\_feature = None

min\_error = float('inf')

for feature in range(n\_features):

if feature in selected\_features:

continue

current\_features = selected\_features + [feature]

current\_error = leave\_one\_out\_error(features[:, current\_features], angles)

if current\_error < min\_error:

min\_error = current\_error

best\_feature = feature

if best\_feature is not None:

selected\_features.append(best\_feature)

errors.append(min\_error)

print(f'Selected feature: {best\_feature}, Current error: {min\_error}')

return selected\_features, errors

# Run the variant of forward selection

selected\_features, errors = forward\_selection\_variant(features, angles)

print(f'Order of features added: {selected\_features}')

print(f'Errors with each number of features: {errors}')

Selected feature: 189, Current error: 310117.257272852

Selected feature: 53, Current error: 186791.79094106288

Selected feature: 137, Current error: 125693.66783699277

Selected feature: 224, Current error: 118213.72795439238

Selected feature: 32, Current error: 108774.43934657286

Selected feature: 24, Current error: 113215.58440938324

Selected feature: 147, Current error: 92182.74733004993

Selected feature: 102, Current error: 83145.94150030859

Selected feature: 13, Current error: 76545.49962277396

Selected feature: 192, Current error: 68555.7720831847

Selected feature: 229, Current error: 67249.20883899964

Selected feature: 138, Current error: 58086.88470231136

Selected feature: 84, Current error: 53196.35343814999

Selected feature: 207, Current error: 49986.69828281485

Selected feature: 206, Current error: 51445.59659133378

Selected feature: 107, Current error: 48746.53651537712

Selected feature: 200, Current error: 50259.34443514915

Selected feature: 180, Current error: 48704.941505726616

Selected feature: 26, Current error: 48156.73466100886

Selected feature: 167, Current error: 46925.03475732019

Selected feature: 116, Current error: 46245.04197879646

Selected feature: 245, Current error: 48130.861018028845

Selected feature: 8, Current error: 48193.85971002209

Selected feature: 132, Current error: 53729.723594547126

Selected feature: 205, Current error: 50890.70962294393

Selected feature: 105, Current error: 51050.84242275881

Selected feature: 80, Current error: 47578.335252495584

Selected feature: 54, Current error: 45523.50821726195

Selected feature: 135, Current error: 44799.7653716227

Selected feature: 125, Current error: 45075.12179828019

Selected feature: 120, Current error: 43192.54833695477

Selected feature: 250, Current error: 44033.613751460674

Selected feature: 174, Current error: 43887.2520916703

Selected feature: 12, Current error: 42049.2783239452

Selected feature: 218, Current error: 41477.711772163224

Selected feature: 46, Current error: 40838.02229442647

Selected feature: 153, Current error: 43306.855498781675

Selected feature: 113, Current error: 41764.96641387828

Selected feature: 56, Current error: 42033.745454403004

Selected feature: 9, Current error: 43568.035766330126

Selected feature: 124, Current error: 43552.35642854261

Selected feature: 208, Current error: 44299.949772008564

Selected feature: 194, Current error: 42597.93211310777

Selected feature: 59, Current error: 42326.068515265084

Selected feature: 228, Current error: 43911.86248493137

Selected feature: 104, Current error: 40814.16862300696

Selected feature: 98, Current error: 40839.95626412244

Selected feature: 89, Current error: 40590.842454710924

Selected feature: 165, Current error: 41833.25715588732

Selected feature: 17, Current error: 40635.943777072556

Selected feature: 166, Current error: 41718.645969139405

Selected feature: 73, Current error: 41618.54961586933

Selected feature: 96, Current error: 39266.60138015826

Selected feature: 18, Current error: 39160.97482155963

Selected feature: 246, Current error: 40692.351181376835

Selected feature: 253, Current error: 41338.94614268295

Selected feature: 142, Current error: 40420.41014823809

Selected feature: 114, Current error: 40769.55147591676

Selected feature: 146, Current error: 41039.54935481189

Selected feature: 81, Current error: 42745.43256443976

Selected feature: 190, Current error: 42267.19069356858

Selected feature: 197, Current error: 42094.67507033936

Selected feature: 214, Current error: 42830.891521044236

Selected feature: 122, Current error: 41858.185701503375

Selected feature: 112, Current error: 43472.93101780308

Selected feature: 71, Current error: 43834.59372763995

Selected feature: 182, Current error: 43923.2140430634

Selected feature: 222, Current error: 47488.811010813595

Selected feature: 101, Current error: 48187.008656656784

Selected feature: 62, Current error: 47318.59504161139

Selected feature: 99, Current error: 47518.989285784664

Selected feature: 184, Current error: 49408.386665803155

Selected feature: 170, Current error: 48591.84066430311

Selected feature: 219, Current error: 48098.428288332776

Selected feature: 179, Current error: 47229.0401529183

Selected feature: 128, Current error: 47229.0401529183

Selected feature: 37, Current error: 49604.56074508418

Selected feature: 57, Current error: 51187.89246598157

Selected feature: 226, Current error: 48849.01319489871

Selected feature: 162, Current error: 48598.107944180985

Selected feature: 87, Current error: 48877.08178097935

Selected feature: 171, Current error: 50375.699931474526

Selected feature: 70, Current error: 50580.904132735704

Selected feature: 188, Current error: 50655.593665749235

Selected feature: 244, Current error: 50246.641874261564

Selected feature: 198, Current error: 48822.33276538652

Selected feature: 49, Current error: 48190.59729954385

Selected feature: 215, Current error: 48793.51233719832

Selected feature: 35, Current error: 48747.7147318507

Selected feature: 15, Current error: 48958.345749860004

Selected feature: 0, Current error: 49727.96256230792

Selected feature: 52, Current error: 51855.077462542264

Selected feature: 151, Current error: 50888.00203703632

Selected feature: 2, Current error: 49351.64265645672

Selected feature: 6, Current error: 50272.658701273525

Selected feature: 100, Current error: 51484.043771094584

Selected feature: 201, Current error: 49294.70832360552

Selected feature: 199, Current error: 49007.74659196625

Selected feature: 181, Current error: 51293.730262507

Selected feature: 115, Current error: 52092.942294369

Selected feature: 133, Current error: 54305.55542715681

Selected feature: 33, Current error: 51782.66409165066

Selected feature: 220, Current error: 53490.961915217435

Selected feature: 251, Current error: 50876.04720105699

Selected feature: 159, Current error: 52820.09360951372

Selected feature: 238, Current error: 51938.20593967722

Selected feature: 230, Current error: 54584.32607301758

Selected feature: 187, Current error: 56147.584059504006

Selected feature: 126, Current error: 57913.49997836994

Selected feature: 106, Current error: 55352.74893485998

Selected feature: 169, Current error: 54158.19204005362

Selected feature: 148, Current error: 53543.058226428744

Selected feature: 68, Current error: 54699.79421140733

Selected feature: 240, Current error: 56077.39575881372

Selected feature: 67, Current error: 54191.93650195313

Selected feature: 58, Current error: 52757.955756556985

Selected feature: 79, Current error: 48824.01348221154

Selected feature: 51, Current error: 52964.96950585049

Selected feature: 93, Current error: 56215.763378311254

Selected feature: 38, Current error: 58890.00855278306

Selected feature: 212, Current error: 66331.47548986146

Selected feature: 97, Current error: 68134.1219821682

Selected feature: 185, Current error: 65108.716556801104

Selected feature: 39, Current error: 63800.53180512514

Selected feature: 186, Current error: 65594.02174033133

Selected feature: 252, Current error: 69836.83233356822

Selected feature: 48, Current error: 69601.5348118147

Selected feature: 191, Current error: 67476.05500370351

Selected feature: 110, Current error: 67555.06321023473

Selected feature: 41, Current error: 67456.41282526619

Selected feature: 195, Current error: 66199.19406495277

Selected feature: 1, Current error: 68337.12435411895

Selected feature: 140, Current error: 68278.45478622302

Selected feature: 233, Current error: 63454.769460520605

Selected feature: 78, Current error: 68780.43454492364

Selected feature: 210, Current error: 67530.66050090811

Selected feature: 241, Current error: 64689.236158483734

Selected feature: 172, Current error: 65668.65074709404

Selected feature: 145, Current error: 66866.5593939598

Selected feature: 163, Current error: 64199.68868961926

Selected feature: 111, Current error: 68724.58989240401

Selected feature: 27, Current error: 70677.6427399893

Selected feature: 16, Current error: 68313.61355255781

Selected feature: 134, Current error: 71314.15829165293

Selected feature: 20, Current error: 68710.76657507585

Selected feature: 247, Current error: 65962.55641056989

Selected feature: 254, Current error: 66910.24044605372

Selected feature: 40, Current error: 71928.37697470676

Selected feature: 85, Current error: 76251.37549565396

Selected feature: 176, Current error: 74970.30845199381

Selected feature: 143, Current error: 75716.56762112003

Selected feature: 150, Current error: 86149.54401667719

Selected feature: 45, Current error: 82592.2650983391

Selected feature: 164, Current error: 87813.49720668049

Selected feature: 5, Current error: 92505.19931515558

Selected feature: 22, Current error: 96546.20746708573

Selected feature: 64, Current error: 92772.79824217819

Selected feature: 248, Current error: 98492.69646381836

Selected feature: 60, Current error: 93686.33589415981

Selected feature: 213, Current error: 82616.4259424722

Selected feature: 3, Current error: 77112.27670186381

Selected feature: 211, Current error: 84335.6281062005

Selected feature: 202, Current error: 88425.73779130157

Selected feature: 155, Current error: 94180.0171529628

Selected feature: 141, Current error: 89906.42350343725

Selected feature: 177, Current error: 90613.61920218173

Selected feature: 42, Current error: 94570.1578621839

Selected feature: 152, Current error: 91843.21011379073

Selected feature: 118, Current error: 95558.97776845784

Selected feature: 77, Current error: 95077.01268124278

Selected feature: 91, Current error: 97663.31280506164

Selected feature: 204, Current error: 99691.30505184202

Selected feature: 76, Current error: 105309.73375109186

Selected feature: 94, Current error: 110224.42774098563

Selected feature: 50, Current error: 110771.03427662884

Selected feature: 61, Current error: 107978.36054004799

Selected feature: 158, Current error: 106803.23494758146

Selected feature: 23, Current error: 111084.84103656132

Selected feature: 29, Current error: 114451.9986984646

Selected feature: 144, Current error: 113117.8530550324

Selected feature: 175, Current error: 116850.73259458723

Selected feature: 65, Current error: 117543.54404876671

Selected feature: 221, Current error: 124738.22935765768

Selected feature: 129, Current error: 121634.87422939415

Selected feature: 117, Current error: 126566.89251714273

Selected feature: 83, Current error: 131824.56598524185

Selected feature: 234, Current error: 137473.89654208883

Selected feature: 131, Current error: 133830.8202857773

Selected feature: 249, Current error: 128896.52949655015

Selected feature: 136, Current error: 128162.37322573506

Selected feature: 44, Current error: 135671.97812138402

Selected feature: 154, Current error: 128358.13232427265

Selected feature: 255, Current error: 128965.40868401145

Selected feature: 69, Current error: 127686.49979838905

Selected feature: 31, Current error: 124413.81909116427

Selected feature: 236, Current error: 125220.2474963744

Selected feature: 47, Current error: 134204.14473212676

Selected feature: 149, Current error: 124611.54410527286

Selected feature: 92, Current error: 124732.45319292939

Selected feature: 217, Current error: 125243.22315049694

Selected feature: 157, Current error: 139574.866724721

Selected feature: 63, Current error: 144952.2629091502

Selected feature: 75, Current error: 146970.80882307419

Selected feature: 7, Current error: 143561.53941140656

Selected feature: 86, Current error: 162956.7659631501

Selected feature: 237, Current error: 156836.32589350542

Selected feature: 103, Current error: 168620.4031482979

Selected feature: 95, Current error: 181974.36228295846

Selected feature: 235, Current error: 164657.3729108529

Selected feature: 183, Current error: 181123.98279589022

Selected feature: 232, Current error: 177089.70828688494

Selected feature: 34, Current error: 175078.80027517723

Selected feature: 231, Current error: 167954.40435708186

Selected feature: 4, Current error: 167675.31042921747

Selected feature: 74, Current error: 174780.44511851098

Selected feature: 139, Current error: 179613.86949156676

Selected feature: 11, Current error: 180444.0327999745

Selected feature: 127, Current error: 184891.75296261997

Selected feature: 223, Current error: 187546.71131189555

Selected feature: 242, Current error: 204751.47374597212

Selected feature: 21, Current error: 203357.27576579488

Selected feature: 225, Current error: 190337.43435331123

Selected feature: 55, Current error: 201762.43782711483

Selected feature: 36, Current error: 198356.04799781484

Selected feature: 178, Current error: 196374.01673301647

Selected feature: 108, Current error: 198555.1105544803

Selected feature: 19, Current error: 208166.69802702375

Selected feature: 90, Current error: 209797.94069751757

Selected feature: 227, Current error: 204442.1077987515

Selected feature: 119, Current error: 227039.85960609082

Selected feature: 243, Current error: 236823.17266327483

Selected feature: 28, Current error: 246208.91291236517

Selected feature: 82, Current error: 235280.33004409206

Selected feature: 160, Current error: 224008.030337183

Selected feature: 30, Current error: 224572.49902487322

Selected feature: 121, Current error: 238773.3572516557

Selected feature: 72, Current error: 248951.13007420246

Selected feature: 66, Current error: 247070.01970181565

Selected feature: 43, Current error: 241934.72054576746

Selected feature: 14, Current error: 252313.5312138403

Selected feature: 203, Current error: 258758.0809408015

Selected feature: 130, Current error: 262349.6134041262

Selected feature: 239, Current error: 287322.8306262382

Selected feature: 123, Current error: 296284.73997282493

Selected feature: 193, Current error: 294167.7421557834

Selected feature: 173, Current error: 312811.26293239585

Selected feature: 25, Current error: 305573.0479343124

Selected feature: 196, Current error: 349832.39081952884

Selected feature: 10, Current error: 361580.46963596524

Selected feature: 168, Current error: 344152.41367836774

Selected feature: 156, Current error: 398915.60100867564

Selected feature: 109, Current error: 441962.20120538597

Selected feature: 216, Current error: 440858.23224591726

Selected feature: 88, Current error: 451606.12871276453

Selected feature: 161, Current error: 442018.16643308965

Selected feature: 209, Current error: 507129.623964493

Order of features added: [189, 53, 137, 224, 32, 24, 147, 102, 13, 192, 229, 138, 84, 207, 206, 107, 200, 180, 26, 167, 116, 245, 8, 132, 205, 105, 80, 54, 135, 125, 120, 250, 174, 12, 218, 46, 153, 113, 56, 9, 124, 208, 194, 59, 228, 104, 98, 89, 165, 17, 166, 73, 96, 18, 246, 253, 142, 114, 146, 81, 190, 197, 214, 122, 112, 71, 182, 222, 101, 62, 99, 184, 170, 219, 179, 128, 37, 57, 226, 162, 87, 171, 70, 188, 244, 198, 49, 215, 35, 15, 0, 52, 151, 2, 6, 100, 201, 199, 181, 115, 133, 33, 220, 251, 159, 238, 230, 187, 126, 106, 169, 148, 68, 240, 67, 58, 79, 51, 93, 38, 212, 97, 185, 39, 186, 252, 48, 191, 110, 41, 195, 1, 140, 233, 78, 210, 241, 172, 145, 163, 111, 27, 16, 134, 20, 247, 254, 40, 85, 176, 143, 150, 45, 164, 5, 22, 64, 248, 60, 213, 3, 211, 202, 155, 141, 177, 42, 152, 118, 77, 91, 204, 76, 94, 50, 61, 158, 23, 29, 144, 175, 65, 221, 129, 117, 83, 234, 131, 249, 136, 44, 154, 255, 69, 31, 236, 47, 149, 92, 217, 157, 63, 75, 7, 86, 237, 103, 95, 235, 183, 232, 34, 231, 4, 74, 139, 11, 127, 223, 242, 21, 225, 55, 36, 178, 108, 19, 90, 227, 119, 243, 28, 82, 160, 30, 121, 72, 66, 43, 14, 203, 130, 239, 123, 193, 173, 25, 196, 10, 168, 156, 109, 216, 88, 161, 209]

Errors with each number of features: [310117.257272852, 186791.79094106288, 125693.66783699277, 118213.72795439238, 108774.43934657286, 113215.58440938324, 92182.74733004993, 83145.94150030859, 76545.49962277396, 68555.7720831847, 67249.20883899964, 58086.88470231136, 53196.35343814999, 49986.69828281485, 51445.59659133378, 48746.53651537712, 50259.34443514915, 48704.941505726616, 48156.73466100886, 46925.03475732019, 46245.04197879646, 48130.861018028845, 48193.85971002209, 53729.723594547126, 50890.70962294393, 51050.84242275881, 47578.335252495584, 45523.50821726195, 44799.7653716227, 45075.12179828019, 43192.54833695477, 44033.613751460674, 43887.2520916703, 42049.2783239452, 41477.711772163224, 40838.02229442647, 43306.855498781675, 41764.96641387828, 42033.745454403004, 43568.035766330126, 43552.35642854261, 44299.949772008564, 42597.93211310777, 42326.068515265084, 43911.86248493137, 40814.16862300696, 40839.95626412244, 40590.842454710924, 41833.25715588732, 40635.943777072556, 41718.645969139405, 41618.54961586933, 39266.60138015826, 39160.97482155963, 40692.351181376835, 41338.94614268295, 40420.41014823809, 40769.55147591676, 41039.54935481189, 42745.43256443976, 42267.19069356858, 42094.67507033936, 42830.891521044236, 41858.185701503375, 43472.93101780308, 43834.59372763995, 43923.2140430634, 47488.811010813595, 48187.008656656784, 47318.59504161139, 47518.989285784664, 49408.386665803155, 48591.84066430311, 48098.428288332776, 47229.0401529183, 47229.0401529183, 49604.56074508418, 51187.89246598157, 48849.01319489871, 48598.107944180985, 48877.08178097935, 50375.699931474526, 50580.904132735704, 50655.593665749235, 50246.641874261564, 48822.33276538652, 48190.59729954385, 48793.51233719832, 48747.7147318507, 48958.345749860004, 49727.96256230792, 51855.077462542264, 50888.00203703632, 49351.64265645672, 50272.658701273525, 51484.043771094584, 49294.70832360552, 49007.74659196625, 51293.730262507, 52092.942294369, 54305.55542715681, 51782.66409165066, 53490.961915217435, 50876.04720105699, 52820.09360951372, 51938.20593967722, 54584.32607301758, 56147.584059504006, 57913.49997836994, 55352.74893485998, 54158.19204005362, 53543.058226428744, 54699.79421140733, 56077.39575881372, 54191.93650195313, 52757.955756556985, 48824.01348221154, 52964.96950585049, 56215.763378311254, 58890.00855278306, 66331.47548986146, 68134.1219821682, 65108.716556801104, 63800.53180512514, 65594.02174033133, 69836.83233356822, 69601.5348118147, 67476.05500370351, 67555.06321023473, 67456.41282526619, 66199.19406495277, 68337.12435411895, 68278.45478622302, 63454.769460520605, 68780.43454492364, 67530.66050090811, 64689.236158483734, 65668.65074709404, 66866.5593939598, 64199.68868961926, 68724.58989240401, 70677.6427399893, 68313.61355255781, 71314.15829165293, 68710.76657507585, 65962.55641056989, 66910.24044605372, 71928.37697470676, 76251.37549565396, 74970.30845199381, 75716.56762112003, 86149.54401667719, 82592.2650983391, 87813.49720668049, 92505.19931515558, 96546.20746708573, 92772.79824217819, 98492.69646381836, 93686.33589415981, 82616.4259424722, 77112.27670186381, 84335.6281062005, 88425.73779130157, 94180.0171529628, 89906.42350343725, 90613.61920218173, 94570.1578621839, 91843.21011379073, 95558.97776845784, 95077.01268124278, 97663.31280506164, 99691.30505184202, 105309.73375109186, 110224.42774098563, 110771.03427662884, 107978.36054004799, 106803.23494758146, 111084.84103656132, 114451.9986984646, 113117.8530550324, 116850.73259458723, 117543.54404876671, 124738.22935765768, 121634.87422939415, 126566.89251714273, 131824.56598524185, 137473.89654208883, 133830.8202857773, 128896.52949655015, 128162.37322573506, 135671.97812138402, 128358.13232427265, 128965.40868401145, 127686.49979838905, 124413.81909116427, 125220.2474963744, 134204.14473212676, 124611.54410527286, 124732.45319292939, 125243.22315049694, 139574.866724721, 144952.2629091502, 146970.80882307419, 143561.53941140656, 162956.7659631501, 156836.32589350542, 168620.4031482979, 181974.36228295846, 164657.3729108529, 181123.98279589022, 177089.70828688494, 175078.80027517723, 167954.40435708186, 167675.31042921747, 174780.44511851098, 179613.86949156676, 180444.0327999745, 184891.75296261997, 187546.71131189555, 204751.47374597212, 203357.27576579488, 190337.43435331123, 201762.43782711483, 198356.04799781484, 196374.01673301647, 198555.1105544803, 208166.69802702375, 209797.94069751757, 204442.1077987515, 227039.85960609082, 236823.17266327483, 246208.91291236517, 235280.33004409206, 224008.030337183, 224572.49902487322, 238773.3572516557, 248951.13007420246, 247070.01970181565, 241934.72054576746, 252313.5312138403, 258758.0809408015, 262349.6134041262, 287322.8306262382, 296284.73997282493, 294167.7421557834, 312811.26293239585, 305573.0479343124, 349832.39081952884, 361580.46963596524, 344152.41367836774, 398915.60100867564, 441962.20120538597, 440858.23224591726, 451606.12871276453, 442018.16643308965, 507129.623964493]

d)

• At some point in the algorithm, from the 6th added feature to approximately the third quarter of the features that the algorithm reach a better solution than in step b). Because the variant of forward selection continues to add features even if they do not improve the performance, which can sometimes lead to better solutions as more features are considered. Therefore the algorithm might find a combination of features that work well together, even if individual features do not show immediate improvement.

• Plot a curve of the performance measure with respect to the number of features.

A graph with orange line

AI-generated content may be incorrect.

**Problem A2:**

1. Discussion on Simple Variable Ranking Methods:

We can use correlation methods such as Pearson correlation which measures the linear relationship between two variables. For the Sculpt Faces data set, we can compute the Pearson correlation coefficient between each feature and each of the target angles (pose and lighting angles). For each feature (pixel), we compute the Pearson correlation coefficient with each of the three target angles, then we can calculate the average the absolute values of these correlations to get a single ranking score for each feature. We can finally rank the features based on their average Pearson correlation scores.

1. In step c) of problem A1, we iteratively added the most useful features using forward feature selection, the order of variable selection is different from the ranking obtained by the leave-one-out error method. Forward selection typically chooses features based on their incremental contribution to reducing prediction error, considering interactions between features. Meanwhile, ranking by correlation treats each feature independently, which may result in a different order.